General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

Friday 23 May 2008 9.00 am to 10.30 am

For this paper you must have:
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The speed, $v\,\mathrm{m\,s^{-1}}$, of a wave travelling along the surface of a sea is believed to depend on

the depth of the sea, d m,
the density of the water, r $\mathrm{kg\,m^{-3}}$,
the acceleration due to gravity, g, and
a dimensionless constant, k

so that

$$v = kd^{a}r^{b}g^{g}$$

where a, b and g are constants.

By using dimensional analysis, show that b $= 0$ and find the values of a and g. *(6 marks)*

2 The unit vectors **i** and **j** are directed due east and due north respectively.

Two runners, Albina and Brian, are running on level parkland with constant velocities of $(5\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$ and $(3\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$ respectively. Initially, the position vectors of Albina and Brian are $(-60\mathbf{i} + 160\mathbf{j})$ m and $(40\mathbf{i} - 90\mathbf{j})$ m respectively, relative to a fixed origin in the parkland.

(a) Write down the velocity of Brian relative to Albina. *(2 marks)*

(b) Find the position vector of Brian relative to Albina t seconds after they leave their initial positions. *(3 marks)*

(c) Hence determine whether Albina and Brian will collide if they continue running with the same velocities. *(3 marks)*

3 A particle of mass 0.2 kg lies at rest on a smooth horizontal table. A horizontal force of magnitude F newtons acts on the particle in a constant direction for 0.1 seconds. At time t seconds,

$$F = 5 \times 10^{3} t^{2}, \quad 0 4 t 4 0.1$$

Find the value of t when the speed of the particle is $2\,\mathrm{m\,s^{-1}}$. *(4 marks)*

4 Two smooth spheres, A and B, have equal radii and masses m and $2m$ respectively. The spheres are moving on a smooth horizontal plane. The sphere A has velocity $(4\mathbf{i} + 3\mathbf{j})$ when it collides with the sphere B which has velocity $(-2\mathbf{i} + 2\mathbf{j})$. After the collision, the velocity of B is $(\mathbf{i} + \mathbf{j})$.

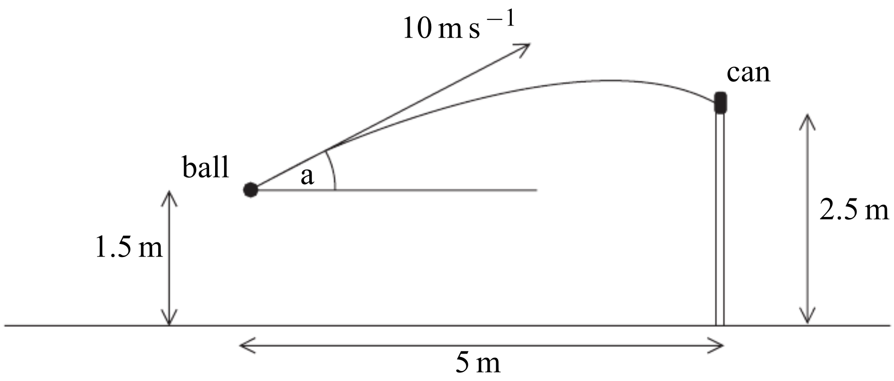
(a) Find the velocity of A immediately after the collision. *(3 marks)*

(b) Find the angle between the velocities of A and B immediately after the collision. *(3 marks)*

(c) Find the impulse exerted by B on A. *(3 marks)*

(d) State, as a vector, the direction of the line of centres of A and B when they collide. *(1 mark)*

5 A boy throws a small ball from a height of 1.5 m above horizontal ground with initial velocity $10\,\text{m s}^{-1}$ at an angle α above the horizontal. The ball hits a small can placed on a vertical wall of height 2.5 m, which is at a horizontal distance of 5 m from the initial position of the ball, as shown in the diagram.



(a) Show that α satisfies the equation

$$49\tan^2\alpha - 200\tan\alpha + 89 = 0$$ *(7 marks)*

(b) Find the **two** possible values of α, giving your answers to the nearest 0.1°. *(3 marks)*

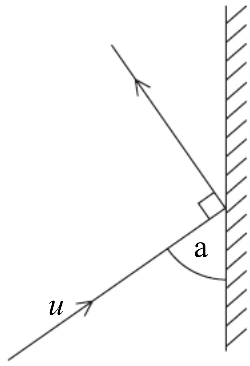
(c) (i) To knock the can off the wall, the horizontal component of the velocity of the ball must be greater than $8\,\text{m s}^{-1}$.

Show that, for one of the possible values of α found in part (b), the can will be knocked off the wall, and for the other, it will **not** be knocked off the wall. *(3 marks)*

(ii) Given that the can is knocked off the wall, find the direction in which the ball is moving as it hits the can. *(4 marks)*

6 A small smooth ball of mass m, moving on a smooth horizontal surface, hits a smooth vertical wall and rebounds. The coefficient of restitution between the wall and the ball is $\frac{3}{4}$.

Immediately before the collision, the ball has velocity u and the angle between the ball's direction of motion and the wall is a . The ball's direction of motion immediately after the collision is at right angles to its direction of motion before the collision, as shown in the diagram.



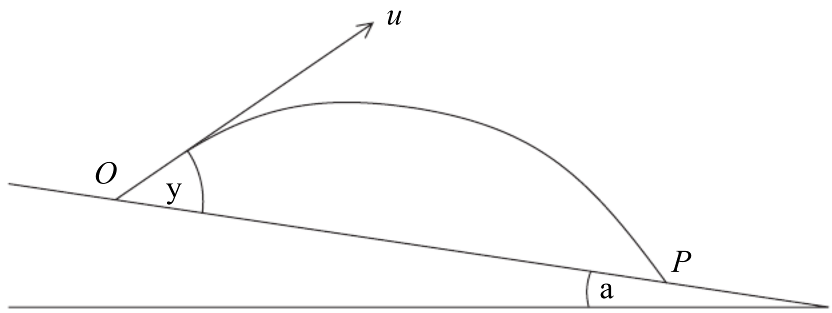
(a) Show that $\tan a = \frac{2}{\sqrt{3}}$. *(5 marks)*

(b) Find, in terms of u, the speed of the ball immediately after the collision. *(2 marks)*

(c) The force exerted on the ball by the wall acts for 0.1 seconds.

Given that $m = 0.2\,\text{kg}$ and $u = 4\,\text{m s}^{-1}$, find the average force exerted by the wall on the ball. *(6 marks)*

7 A projectile is fired with speed u from a point O on a plane which is inclined at an angle a to the horizontal. The projectile is fired at an angle y to the inclined plane and moves in a vertical plane through a line of greatest slope of the inclined plane. The projectile lands at a point P, lower down the inclined plane, as shown in the diagram.



(a) Find, in terms of u, g, y and a, the greatest perpendicular distance of the projectile from the plane. *(4 marks)*

(b) (i) Find, in terms of u, g, y and a, the time of flight from O to P. *(2 marks)*

(ii) By using the identity $\cos A \cos B + \sin A \sin B = \cos(A - B)$, show that the distance OP is given by $\dfrac{2u^2 \sin \mathrm{y} \cos(\mathrm{y} - \mathrm{a})}{g \cos^2 \mathrm{a}}$. *(6 marks)*

(iii) Hence, by using the identity $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ or otherwise, show that, as y varies, the maximum possible distance OP is $\dfrac{u^2}{g(1 - \sin \mathrm{a})}$. *(5 marks)*

END OF QUESTIONS

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